Exercise 19

Evaluate the integral.

$$\int_1^3 (x^2 + 2x - 4)\,dx$$

Solution

According to part 2 of the fundamental theorem of calculus,

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where F is an antiderivative of f. Use the properties of integrals given at the bottom of page 385 to simplify the integral before using this theorem.

$$\begin{aligned}
\int_1^3 (x^2 + 2x - 4)\,dx &= \int_1^3 x^2\,dx + \int_1^3 2x\,dx - \int_1^3 4\,dx \\
&= \int_1^3 x^2\,dx + 2\int_1^3 x\,dx - 4\int_1^3 1\,dx \\
&= \left(\frac{x^3}{3}\right)\Bigg|_1^3 + 2\left(\frac{x^2}{2}\right)\Bigg|_1^3 - 4\,(x)\Big|_1^3 \\
&= \left(\frac{3^3}{3} - \frac{1^3}{3}\right) + 2\left(\frac{3^2}{2} - \frac{1^2}{2}\right) - 4(3-1) \\
&= \frac{26}{3} + 2(4) - 4(2) \\
&= \frac{26}{3}
\end{aligned}$$